

Estimation of uniaxial stress component in NaCl samples compressed in a tungsten carbide anvil high-pressure x-ray apparatus

A. K. Singh* and G. C. Kennedy

Institute of Geophysics and Planetary Physics, University of California, Los Angeles, California 90024

(Received 7 August 1975; in final form 26 March 1976)

The uniaxial stress component (USC) in NaCl samples compressed in a tungsten carbide opposed anvil high-pressure x-ray apparatus has been determined to 10 GPa by a method proposed earlier by Singh and Kennedy. In the pressure range 1–10 GPa, the pressure dependence of the lower estimates of USC can be expressed by the relation $t(p) = 0.24(\pm 0.04) + 0.11(\pm 0.005)p$, where $t(p)$ is the USC at a pressure p , $t(p)$ and p being expressed in the units of gigaPascals (GPa). These results have been used to estimate the difference, ΔP , between the pressures calculated from the hydrostatic component of strain and the strain averaged for the first six strong reflections of NaCl. ΔP is found to increase linearly from ~ 0.1 GPa at 1 GPa to ~ 0.4 GPa at 10 GPa.

PACS numbers: 62.50.+p, 61.10.Fr, 62.20.Fe

INTRODUCTION

The tungsten carbide opposed anvil x-ray cameras¹⁻³ have been extensively used in the x-ray diffraction studies of solids at high pressures. The sample is placed in a small hole at the center of a boron (or boron-epoxy) disk and pressed between the anvils. A collimated x-ray beam is passed perpendicular to the loading direction and the diffraction pattern is recorded on a photographic film. The data are analyzed on the assumption that the pressure on the sample is hydrostatic. In practice, considerable departure from the hydrostaticity of the sample pressure is caused by the facts that the sample is pressurized by the application of an uniaxial load and that the solids have appreciable yield strength. In such cases, the sample pressure is better described as a superposition of a hydrostatic and an uniaxial stress component. In a previous paper,⁴ a method was proposed to estimate the uniaxial stress component, and this has been successfully applied to the diffraction data for Si and NaCl. In this paper, the variation of uniaxial stress component in NaCl with pressure has been studied. NaCl was chosen for this investigation because it is often used as a pressure-transmitting medium and as an internal pressure standard in high-pressure x-ray investigations. A study of the variation of the uniaxial stress with pressure will be useful in evaluating the performance of NaCl as a pressure transmitter.

THEORY

As was shown in a previous paper,⁴ the lattice strain in a direction (hkl) for a sample belonging to the cubic system is given by

$$\epsilon_w(hkl) = [d_w(hkl) - d_0(hkl)]/d_0(hkl) \quad (1)$$

$$= m(p)\Gamma(hkl) + C(p), \quad (2)$$

where

$$m(p) = \alpha[S_{11}(p) - S_{12}(p) - \frac{1}{2}S_{44}(p)]t(p) \quad (3)$$

$$= \epsilon_w(p) + \alpha S_{12}(p)t(p) - \beta t(p)[C_{11}(p) + 4C_{12}(p) - 2C_{44}(p)] \\ \times \{2[C_{11}(p) + 2C_{12}(p)][C_{11}(p) - C_{12}(p) + 3C_{44}(p)]\}^{-1}, \quad (4)$$

where $\epsilon_w(p)$ is the strain corresponding to the hydrostatic component p , $t(p)$ is the uniaxial stress component, $S_{ij}(p)$ and $C_{ij}(p)$ are, respectively, the single-crystal elastic compliances and elastic moduli, and $\Gamma(hkl) = (h^2k^2 + k^2l^2 + l^2h^2)/(h^2 + k^2 + l^2)^2$. If the state of stress continuity in the crystallites is assumed, then $\alpha = 1$ and $\beta = 0$, and $\alpha = 0$ and $\beta = 1$ if the state of strain continuity is assumed. For an intermediate state $0 < \alpha$, $\beta > 1$.

A plot of $\epsilon_w(hkl)$ versus $\Gamma(hkl)$ is a straight line. The magnitude of $t(p)$ can be estimated from the slope of the line if $S_{ij}(p)$ are known. It must be emphasized that this method fails if either $\alpha = 0$ or $S_{11}(p) - S_{12}(p) - \frac{1}{2}S_{44}(p) = 0$. In the case of NaCl, $S_{11}(p) - S_{12}(p) - \frac{1}{2}S_{44}(p) \neq 0$, and experimentally it is found that $m(p) \neq 0$, which indicates that both α and $t(p)$ are nonzero.

III. EXPERIMENTAL WORK

The opposed anvil apparatus used in this study was similar to that described by McWhan and Bond.³ The samples were prepared by grinding 99.9% pure NaCl from J. T. Baker Chemical Co. The powdered sample was placed in a 0.25-mm hole at the center of a boron-epoxy disk 3.2 mm in diameter and 0.5 mm thick. The boron-epoxy disk with the sample at the center was placed between the anvils such that the sample was on the axis of the anvils. The anvils were made of tungsten carbide (carbide grade 883) and the tips were in the form of truncated cones with a 3.2-mm flat face and 85° half-cone angle. The load was applied with a Black Hawk 7-ton jack. A well-collimated beam of Mo radiation filtered by Zr foil was directed at the sample. The diffraction pattern was recorded on a cylindrical film cassette of radius 57.3 mm. The diffraction patterns were occasionally recorded on a two-film cassette⁵ to cross-check the results. For each sample the diffraction pattern was recorded with the sample under pressure; a second pattern was recorded after releasing the pressure to 1 atm. A fresh boron-epoxy disk was used after each run. After each run, the centering of the sample was checked under a microscope. The displacement of the sample after application of pressure was observed quite infrequently; only two out of a total of 1

TABLE I. The various parameters derived from the measurement of lattice parameters of NaCl at high pressure. S_e , S_m , and S_c denote, respectively, the standard deviations of ϵ , m , and C .

Run	$m(p) \times 10^2$	$-C(p) \times 10^2$	$S_e \times 10^2$	$S_m \times 10^2$	$S_c \times 10^2$	$\epsilon_w(p) \times 10^2$	$-t(p) \text{ (GPa)}^a$
A-10	0.788	3.325	0.11	0.34	0.07	3.409	0.375
B-6	0.217	1.138	0.09	0.29	0.06	1.185	0.135
B-9	0.590	1.738	0.11	0.35	0.07	1.839	0.331
B-14	0.654	3.414	0.11	0.35	0.07	3.482	0.309
B-19	1.042	4.636	0.19	0.61	0.12	4.716	0.464
C-12	0.411	2.909	0.08	0.25	0.05	2.958	0.202
D-10	0.607	2.085	0.07	0.21	0.04	2.178	0.325
D-15	0.474	3.446	0.10	0.32	0.06	3.495	0.224
D-20	0.202	3.668	0.11	0.35	0.07	3.688	0.094
E-10	0.353	1.880	0.09	0.30	0.06	1.938	0.194
G-25	0.833	5.861	0.11	0.35	0.07	5.912	0.361
H-30	0.480	6.984	0.12	0.38	0.08	7.006	0.210
I-25	0.432	5.707	0.14	0.45	0.09	5.735	0.188
J-30	0.900	7.409	0.16	0.50	0.10	7.449	0.396
K-10	0.393	2.044	0.06	0.18	0.04	2.105	0.211
L-30	0.758	6.402	0.13	0.40	0.09	6.444	0.340
M-20	1.199	4.306	0.13	0.43	0.09	4.404	0.540

^aNegative sign indicates that $t(p)$ is compressive.

runs showed sample displacement in excess of 0.02 mm, and were discarded. The exposure time with a Machlett x-ray tube operated at 45 kV and 18 MA ranged from 6 h for low sample pressures to 12 h for the high (~10 GPa) sample pressures. In the entire pressure range (~1–10 GPa) it was possible to record 200, 220, 222, 400, 420, and 422 reflections.

IV. RESULTS AND DISCUSSIONS

The lattice strains $\epsilon_w(hkl)$ for 200, 220, 222, 400, 420, and 422 reflections were calculated from the measurements on the patterns recorded with the sample at high pressure and the corresponding patterns taken with the sample at atmospheric pressure. Equation (2) was fitted to the $\epsilon_w(hkl)$ data by the method of least squares. The slope $m(p)$, and the intercept on the $\epsilon_w(hkl)$ axis, $C(p)$, obtained from 17 different runs are listed in Table I. A plot of $m(p)$ against $C(p)$, and also p , is shown in Fig. 1.

The estimates of $t(p)$ can be made from the knowledge of $S_{ij}(p)$ and $m(p)$. To calculate $S_{ij}(p)$ at a given pressure p , first $C_{11}(p)$, $C_{12}(p)$, and $C_{44}(p)$ were evaluated using the values of $\partial C_{11}/\partial p$, $\partial C_{12}/\partial p$, and $\partial C_{44}/\partial p$ obtained by Bartels and Schuele.⁶ $S_{11}(p)$, $S_{12}(p)$, and $S_{44}(p)$ were then calculated by using the well-known relations among the elastic compliances and the elastic moduli. It must be mentioned that only the first derivatives of the elastic moduli were used in the evaluation of $S_{ij}(p)$. The values of $S_{ij}(p)$, especially in the high-pressure range, might be in error because of the neglect of the higher-order pressure derivatives of the elastic moduli. In a recent study, Spetzler, Sammis, and O'Connell⁷ reported the second derivatives of the various elastic moduli of NaCl. However, their data suggest maxima in the C_{ij}^S -vs- p curves occurring at 7.4, 4.8, and 4.1 GPa, respectively, for C_{11}^S , C_{12}^S , and C_{44}^S . Above these pressures the elastic moduli decrease with increasing pressure. A maximum in the C_{ij}^S -vs- p curve is expected since the first and the second derivatives of C_{ij} have opposite signs. However, occurrence of maxima at

rather low pressures indicates that either the values of the second derivatives are too large due to experimental errors or the third and higher derivatives become appreciable above ~4 GPa. Because of this uncertainty the results of Spetzler, Sammis, and O'Connell⁷ were not used in the present case. Using values of $S_{ij}(p)$ estimated from the data of Bartels and Schuele,⁶ the lower estimates of $t(p)$ were obtained. From the knowledge of $S_{12}(p)$ and $t(p)$, $\epsilon_w(p)$ values were calculated for each run using relation (4) with $\beta=0$ and $\alpha=1$. The values of $t(p)$ and $\epsilon_w(p)$ for each run are listed in Table I. A plot of $t(p)$ versus p is shown in Fig. 2. The pressure derivative of $t(p)$ was determined by fitting an equation of the form

$$t(p) = t(0) + t'(0)p$$

by the method of least squares. The values of $t(0)$ and $t'(0)$ were found to be, respectively, 0.24 ± 0.04 GPa and 0.011 ± 0.005 , the errors indicated being the probable errors.

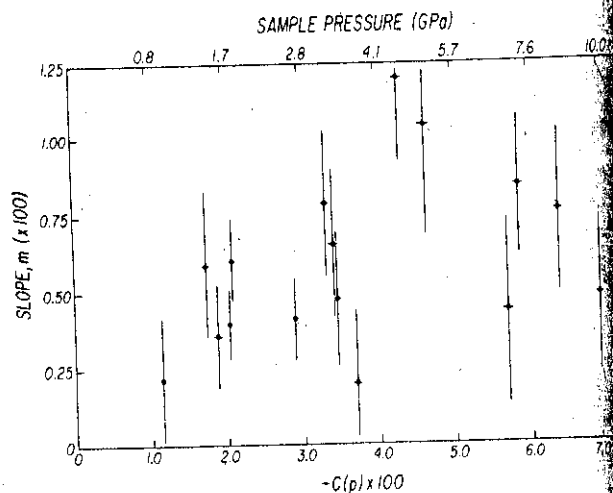


FIG. 1. The slope of the $\epsilon_w(hkl)$ -vs- $p(hkl)$ line as a function of pressure.

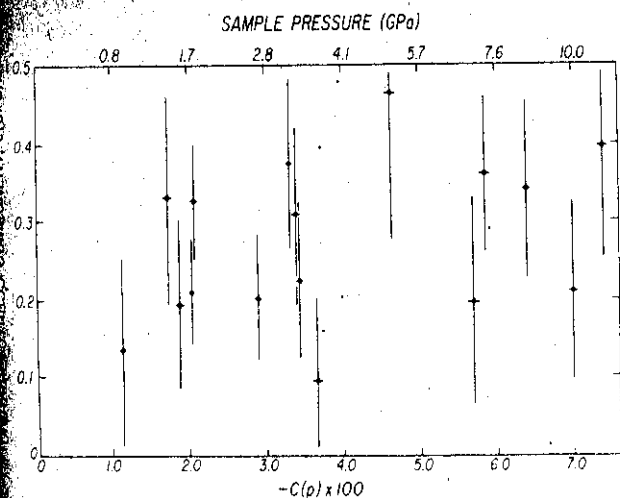


FIG. 2. The variation of the uniaxial stress component with pressure. The uniaxial stress component is calculated with $\bar{\epsilon} = 1$.

The x-ray diffraction experiments have been done carefully to minimize the various errors, and the x-ray data are of highest quality that can be obtained from such an experimental setup. In deriving $m(p)$ from plots such as that shown in Fig. 1, we are making use of small differences in strains which depend on (hkl) . This leads to a large scatter in the determination of $m(p)$. In the following paragraphs a brief discussion of the errors in measurement is given.

The random errors in the measurement arise mainly from the errors in reading the film. An analysis of the data indicates a standard deviation of 0.03 mm in the measurement of the diffraction ring diameters. This amounts to a standard deviation of 0.007° in the determination of θ angles. The standard deviation in the determination of strains, ϵ , is angle dependent and ranges from 0.001 for (200) reflection to 0.0003 for (422) reflection.

The departures of the actual experimental condition from the ideal Debye-Scherrer geometry lead to systematic errors.⁸ These errors are dependent on the θ angle. In the present case, the most important source of error is the displacement of the sample from the center of the goniometer face. As stated earlier, the runs showing a sample shift of more than 0.02 mm were discarded. The estimated error in the θ angle is less than 0.005° . The absorption error is small for NaCl when Mo radiation is used. It must be emphasized that all the systematic errors are functions of θ , and since there is no correlation between $\Gamma(hkl)$ and θ , these systematic errors will appear as random errors in a $\epsilon(hkl)$ -vs- $\Gamma(hkl)$ plot. The standard deviations listed in Table I are derived from the least-squares fit of the $\epsilon(hkl)$ -vs- $\Gamma(hkl)$ plot, and therefore include both the random and the θ -dependent systematic errors. In deriving $t(p)$ from the $m(p)$ values, $S_{ij}(p)$ values are needed. Any error in $S_{ij}(p)$ values will lead to a systematic error in $t(p)$. The error from this source alone is nearly 1%, and is negligible as compared to the errors of measurement.

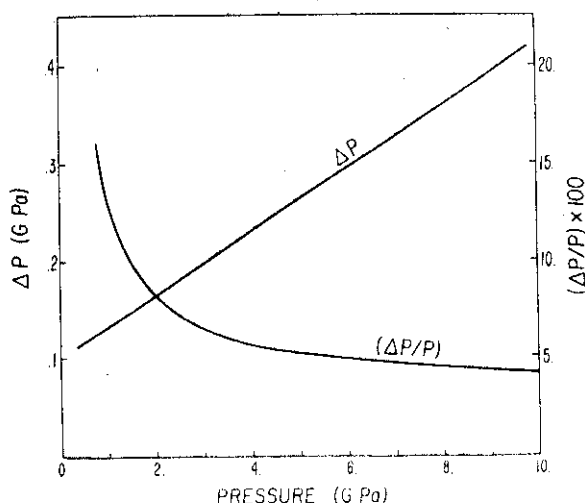


FIG. 3. ΔP and $\Delta P/P$ as a function of P . ΔP is the difference between the pressure calculated from $\epsilon(p)$ and $\bar{\epsilon}(hkl)$.

Recently, Kinsland and Bassett⁹ attempted to measure the USC in NaCl and MgO by measuring the ellipticity of the x-ray diffraction rings¹⁰ in a diamond cell. These measurements indicate the presence of large USC in MgO, but not in NaCl. Most probably the ellipticity in the diffraction rings caused by USC in NaCl is smaller than the error in their experiments.⁹

The uniaxial stress component is essentially the difference between the axial and the radial stress components. From simple arguments^{4,11} it can be shown that the difference between the axial and the radial stress components is equal to the flow stress of the sample material. The flow stress of a material is very sensitive to the grain size and the strain history of the sample. As a result, the USC also will depend on the grain size and the strain history of the sample. Often, the USC measured on two batches of samples differed greatly. For example, in a previous study⁴ a value of 0.55 GPa was obtained for USC at $C(p) = 0.01$. In the present set of measurements an average value of 0.25 GPa is obtained. The present set of measurements were made on the specimen from a single batch of powdered NaCl. The average grain size was 0.02 mm. However, the effect of the grain size and the strain history on USC is prominent only at low pressures. At high pressures the deformation of the sample is severe, and produces a grain structure which is nearly independent of the initial grain size.

The strain $\epsilon(hkl)$ as calculated from Eq. (1) is less than the hydrostatic component $\epsilon(p)$, since $\epsilon(hkl)$ and $\epsilon(p)$ are negative while the term containing $t(p)$ in Eq. (2) is positive. NaCl is often used as a pressure standard. The pressure is calculated from the knowledge of the equation of state of NaCl and the lattice strain averaged over all the observed reflections. The average lattice strain $\bar{\epsilon}(hkl)$ is given by

$$\bar{\epsilon}(hkl) = \epsilon(p) + S_{12}(p)t(p) + [S_{11}(p) - S_{12}(p) - \frac{1}{2}S_{44}(p)]t(p)\bar{\Gamma}(hkl).$$

$\bar{\Gamma}(hkl)$ is the average of $\Gamma(hkl)$ for the line used in the calculation of $\bar{\epsilon}(hkl)$. In practice, 200, 220, 400, 420, and 422 lines are easily observed. $\bar{\Gamma}(hkl)$ for these lines is 0.1656. The difference, ΔP , between the pressure calculated from $\epsilon(p)$ and the pressure calculated from $\bar{\epsilon}(hkl)$ has been estimated using Decker's equation of state,¹² and is shown in Fig. 3. It is seen that ΔP increases almost linearly from ~ 0.1 GPa at 1 GPa to ~ 0.4 GPa at 10 GPa.

The results reported here correspond to a tungsten carbide opposed anvil setup, a boron-epoxy gasket of the dimensions stated earlier, and an initial average grain size of 0.02 mm of the sample material. It can be easily shown^{4,11} that the USC is equal to the flow stress of the sample material, and therefore should depend only on the grain size and the pressure of the sample. Thus, one would expect the magnitude of USC to be independent of the experimental parameters, such as the dimensions of the anvil faces and of the boron-epoxy gasket. Further, the experiments with ungasketed opposed anvil devices^{9,10} should also give results in agreement with the results from gasketed experiments. However, until more extensive measurements with different size gaskets and also with ungasketed apparatus are available to support this point, it is desirable to bear in mind the experimental conditions under which these results are obtained.

ACKNOWLEDGMENTS

The authors wish to thank Professor J. C. Jamieson for his comments on the manuscript. The financial support from Lawrence Livermore Laboratory (6376205) is acknowledged.

*On leave from the Materials Science Division, National Aeronautical Laboratory, Bangalore-17, India.

¹J. C. Jamieson and A. W. Lawson, *J. Appl. Phys.* **33**, 776 (1962).

²E. A. Perez-Albuern, K. F. Forsgren, and H. G. Drickamer, *Rev. Sci. Instrum.* **34**, 29 (1964).

³D. B. McWhan and W. L. Bond, *Rev. Sci. Instrum.* **35**, 626 (1964).

⁴A. K. Singh and G. C. Kennedy, *J. Appl. Phys.* **45**, 4686 (1974).

⁵A. K. Singh, *Rev. Sci. Instrum.* **43**, 1311 (1972).

⁶R. A. Bartels and D. E. Schuele, *J. Phys. Chem. Solids*, **26**, 537 (1965).

⁷H. Spetzler, C. G. Sammis, and R. J. O'Connell, *J. Phys. Chem. Solids* **33**, 1727 (1972).

⁸H. P. Klug and L. E. Alexander, *X-ray Diffraction Procedures* (Wiley, New York, 1967).

⁹G. L. Kinsland and W. A. Bassett, *Rev. Sci. Instrum.* **47**, 130 (1976).

¹⁰Weaver J. Scott, Taro Takahashi, and William A. Bassett, *Trans. Am. Geophys. Union* **53**, No. 4 (1972).

¹¹A. L. Ruoff, *J. Appl. Phys.* **45**, 1389 (1975).

¹²D. L. Decker, *J. Appl. Phys.* **42**, 3239 (1971).